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QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

Professor Dresden discusses the minimum length from a fixed point to a fixed line, measured along a circular arc tangent to another fixed line through the given point. His result may be shown to be geometrically equivalent to the following (stated in Professor Dresden's notation): the minimum length is such that it is equal to the length of that segment of the tangent to the circle at its intersection with the line $x = b$ which falls between that line and the y-axis. Is there a way of seeing directly without the use of analysis that such an arc exists and furnishes a minimum?

Mr. Anning points out the faulty treatment of mathematical topics in general works of reference. The two works to which he refers are of course not isolated instances; in fact it is not improbable that they are among the least culpable. From three other well-known reference compendia the following excerpts from the article *Hyperbola* are chosen:

> "Analytically the hyperbola is the locus of a point which moves so that. . . ."
>
> "If two similar cones be placed apex to apex, and with the lines joining the apex and center of base in each in a straight line, then if a plane which does not pass through the apex be made to cut both cones each of the sections will be a hyperbola. . . . If the axes of coördinates be turned at right angles to their former position, two additional curves will be formed. . . . These two are called *conjugate hyperbolas,* and have the same asymptotes as the original hyperbolas."
>
> "The straight lines . . . known as asymptotes gradually approach the curve, but actually only meet it at points infinitely distant."

Perhaps we may draw the moral that no work intended to cover the whole field of knowledge, however eminent its editorial staff, should ever be trusted for precise information on a technical subject.

The deluge of geometric proofs of the law of tangents (*1920,* 53, 465; *1921,* 71, 170) led the editor to suggest terminating discussion on this subject. Two proofs which were received before the publication of this recommendation appear below. That by Professor Bradley is in fact somewhat different in principle from the usual proofs. That by Mr. Yamanouti is closely related to the usual proofs, but cannot be read out of Professor Lovitt's consolidated figure (*1920,* 465). Finally, Professor Lovitt shows how a figure similar to that used by Mr. Yamanouti, but more general, may be utilized to prove a theorem which includes the law of tangents as a special case. Historical and bibliographical notes are added by Professor Archibald.

Similar in character is the discussion of Professor Rusk, in which a number of familiar formulæ of trigonometry are considered with regard to their deducibility from the sine, cosine, and projection formulæ.

Professor Uhler gives an account of the method he uses to make the ordinary multiplication of two large numbers convenient and fool-proof. The actual work for the multiplication of e^{π} and $e^{-\pi}$ to more than fifty places is reproduced.

Mr. Haldeman shows how a certain quartic curve may be used in order to construct a regular heptagon. Compare a similar article by the same author (*1919*, 390), in which the construction is effected by means of an equilateral hyperbola. Reference may be made also to Note 24 in the department of Problems and Solutions elsewhere in this issue.

I. The Shortest Circular Path from a Point to a Line.

By Arnold Dresden, University of Wisconsin.

Introduction. In connection with the study of a certain type of problem in the calculus of variations, I was led to consider the following question:

"Let there be given a line $x = b$ and a system of circles tangent at the origin to a line $y = mx$ $(m > 0)$ (see figure 1). We take on each of these circles the arc from the origin to the nearer of the two points of intersection with the line $x = b$ and inquire which among the circles of the system furnishes for this arc the minimum value; that is, we inquire which among the circular arcs passing through the origin with slope m will furnish the shortest path from the origin to the line $x = b$."

The intuitive reply to this question was that the shortest circular path is furnished by the circle which cuts the line $x = b$ at right angles. This reply proved to be incorrect. It seemed therefore worth while to publish a discussion of this question which incidentally yielded other points of interest.

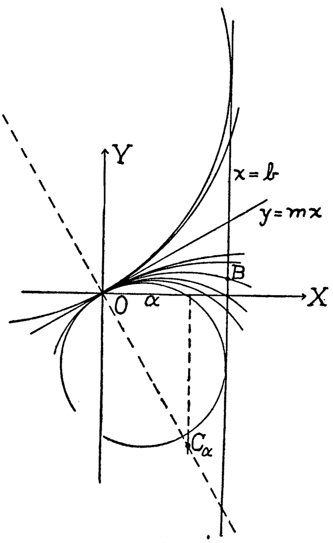

FIG. 1.

1. The circles of the system are given by the equation

$$(x - \alpha)^2 + \left(y + \frac{\alpha}{m}\right)^2 = \frac{\alpha^2(m^2 + 1)}{m^2}, \tag{1}$$

the parameter α being the x-coördinate of the center. The points of intersection of this circle with the line $x = b$ are real if $\alpha \geqq b/(1 + \mu)$ or $\alpha \leqq b/(1 - \mu)$, where

$$\mu = \frac{\sqrt{1 + m^2}}{m} = \operatorname{cosec} \phi,$$

$(\phi < \pi/2)$ being the inclination of the line $y = mx$. The length of the arc OB then becomes

$$L(\alpha) = \int_0^b \sqrt{1 + y'^2}\, dx = \alpha\mu \int_0^b \frac{dx}{\sqrt{\alpha^2\mu^2 - (x - \alpha)^2}} = \alpha\mu \left\{ \arcsin \frac{b - \alpha}{\alpha\mu} + \arcsin \frac{1}{\mu} \right\},$$

this function being defined only for values of α greater than $b/(1 + \mu)$ or less than $b/(1 - \mu)$. The question we raised is that of determining the value of α for which $L(\alpha)$ is a minimum.